Decision Analysis

What makes a decision hard to make?

- **Competing objectives**
- **Uncertainty**
- **Politics**
- **Lack of information**
- **Differing perspectives of Decision makers**
- **Disagreement over what is to be accomplished**
- **Complexity**

Note: Some of these overlap

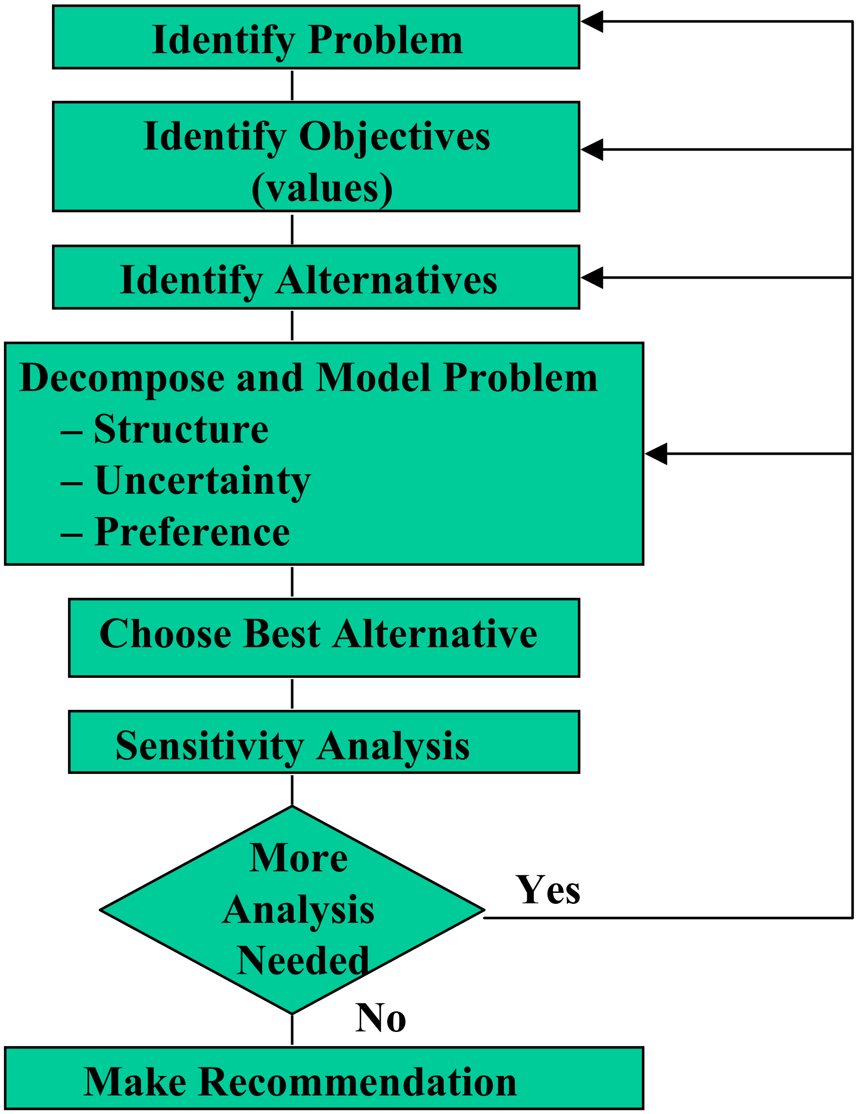
Decision Analysis

What is decision analysis?

A Method to:

- **Organize or structure complex problems for analysis**
- **Deal with tradeoffs between multiple objectives**
- **Identify and quantify sources of *uncertainty***
- **Incorporate subjective judgments**

Decision Analysis Process



Value Focused Thinking

- **Values are fundamental to all we do**
- **Values <u>should</u> drive decisions**
- **Alternative-Focused Thinking**
  - **Standard Approach**
    - **Problem arises**
    - **Problem solving begins**
    - **Alternative solutions listed**
    - **Best alternative chosen**
  - **Limited way to think**
    - **Reactive not proactive**
    - **Incomplete analysis**
    - **Backward thinking**

Evaluating Alternatives - Value Models

- **Objectives related to alternatives by *Attributes***
- **Attributes are measures of achievement of objectives**
  - **Quantitative**
  - **Reflect consequences**
- **Can be one or many (both objectives and attributes)**
  - **One is simple and unusual**
  - **Multiple attributes are more common**
  - **Multiple attributes more difficult to handle**

Value Models

- **Notation:**

A_j, j = 1, . . ., J is the set of alternatives

O_i, i = 1, . . ., n is the set of objectives

X_i, i = 1, . . ., n is the set of measures or attributes

– **Note: attribute X_i is associated with objective O_i**

x_i is the outcome of attribute X_i

– **<u>Example</u>: O_1 is the objective "Maximize Profit"**

X_1 is "the profit in $ millions" with x_1 being $168 million

– **$x = (x_1, x_2, . . ., x_n)$ is the expected consequences of alternative**

Value Model

- a.k.a. "Objective Function"
- Denoted v(x)
- Assigns a number to the consequences, x, of an alternative
- Used to determine preference among alternatives
- Types of value functions
  - Ordinal value function - ranking only
  - Measurable value function - strength of preference
  - Utility function - indicates uncertainty
- Form depends on relationships between attributes
- Convention: $v(x^A) > v(x^B)$ iff we prefer alt A to alt B

Relations Between Attributes

- **Attributes will be added together to get value of alternative**
- **Attributes must not overlap**
- **Must avoid double counting**
- **Attributes must be *independent* of each other in some sense**
- **There is more than one type of independence**

Structures of Value Functions

Additive Value Function

$$v(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} k_i v_i(x_i)$$

where k_i is a positive scaling constant

v and v_i are value functions scaled from 0 to 1

★ **All combinations of attributes are Preferentially Independent!**

Structures of Value Functions

Multiplicative Value Functions

$$1 + kv(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} \left[1 + kk_i v_i(x_i)\right]$$

★ **All pairs of attributes are Weak-Difference Independent**

– **Example:**

$$v(x_1, x_2) = k_1 v_1(x_1) + k_2 v_2(x_2) + k_3 v_1(x_1) v_2(x_2)$$

– **Looks like regression equation with interactions**

– **More difficult to use**

➡ **<u>Bottom Line</u>: Use Additive Independent attributes!**

Example - Decision Problem

Which location should be chosen for the new factory?

<u>Procedure</u>:

(0) Choose Fundamental Objective

- **Profitable enterprise in unpenetrated market**

(1) Determine important Means Objectives

- **Markets**
- **Availability of raw materials and supplies**
- **Taxes**
- **Availability of labor**
- **Cost of labor**

Example - Decision Problem (continued)

(2) Find ways of measuring the objectives

- **Markets -** ***Median family income***
- **Availability of raw materials and supplies**
  - ***Sufficient or not***
- **Taxes -** ***Corporate tax rate***
- **Availability of labor -** ***Population***
- **Cost of labor -** ***Median hourly wage***

Example - Decision Problem (continued)

(3) Scaling objective measures

- **Want each objective measured on scale of 0 to 1**
  - **0 means worst**
  - **1 means best**
  - **others measured proportionally**

x is raw score →

$$S_{other}(x) = \frac{x - worst}{best - worst}$$

- **Limitation: proportional score means "Risk Neutrality"**
  - **Ignores riskiness of alternatives**
  - **Important when considering uncertainty**

Example - Decision Problem (continued)

(4) Weighting the objectives (criteria)

<u>Answer the question</u>: How do we tradeoff increased value in one objective for lower value on the others?

- **Methods:**
  - **- Pricing out**
  - **- Swing weights ***
  - **- Lottery weights**
- **Often totally subjective**
- **Avoid “Importance Weights”**
- **Should be based on range of variation of attributes**

Swing Weights

- **Based on comparing ranges of variation of attributes**
- **Can be used for non-quantitative attributes**
- **Method:**

(1) Find "Worst Conceivable Alternative"

- **Lowest score in each attribute**
- **May be imaginary**

(2) Pick attribute that gives greatest improvement when "swings" to highest level - remember increase

(3) Pick attribute that gives next highest increase when swung

- **by percentage - how does it compare with the first?**
- **never greater than 1 since first is best**

Swing Weights (continued)

(4) Repeat for rest of attributes

(5) Assess weights by noting that:

$$\left.\begin{aligned} k_2 &= p_{21}k_1 \\ k_3 &= p_{31}k_1 \\ &\vdots \\ k_n &= p_{n1}k_1 \\ \sum_{i=1}^{n} k_i &= 1 \end{aligned}\right\}$$

– Solve for the k's

where k_i = weight of attribute i,

and p_{i1} = percentage of improvement compared with attribute 1

Decision Trees

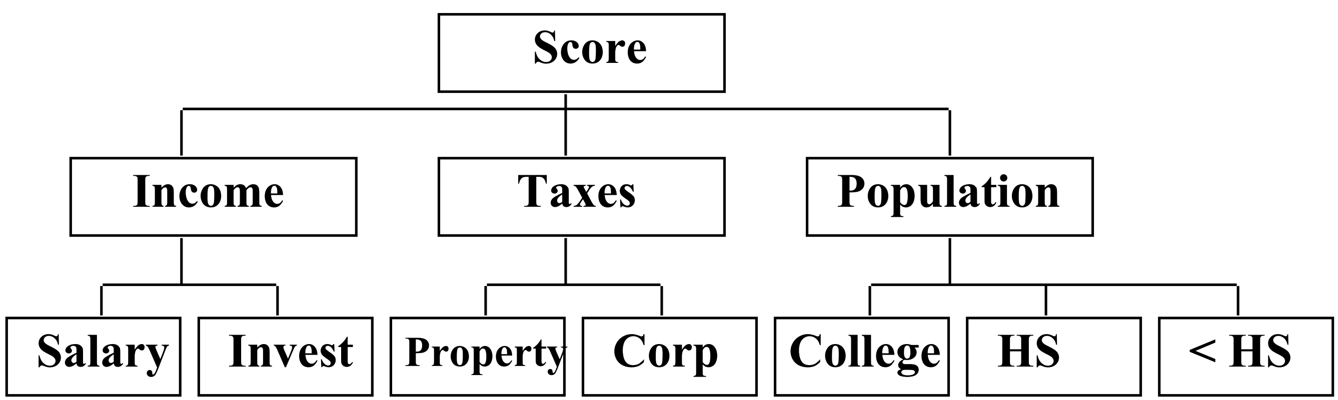
Interpretation 1 – Extension of the Value Function

- **We have:**

$$V = k_1 u_1 (x_1) + k_2 u_2 (x_2) + \ldots + k_n u_n (x_n)$$

– an additive value function

- **Suppose x_i for some i in {1,2, ... ,n} is the value from a value function itself**
- **Gives tree structure or hierarchy**
- **Example:**



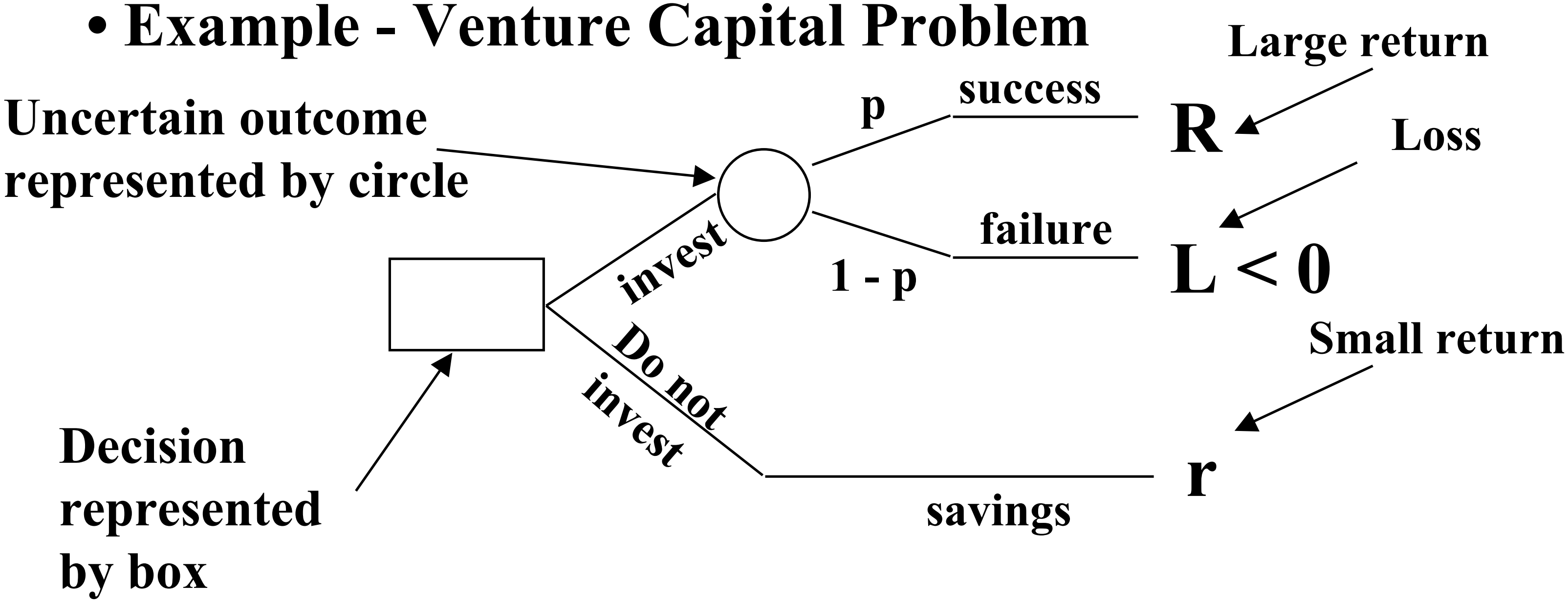
Decision Trees (continued)

- **Each level has its own weights**
  - **Weights sum to 1**
  - **Weights correspond to relative importance of the criteria**
  - **Weights assessed as in other additive value function**
    - **Swing weights**
    - **Lotteries**
    - **Pricing out**
    - **Analytical Hierarchy Process (AHP)** ***in book***
- **Each additive value function is independent of the others**

Decision Trees

Interpretation 2 – Uncertainty in Outcomes

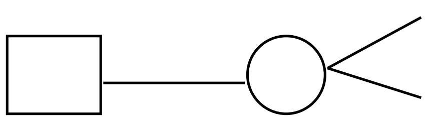
- Probabilities replace the weights
  - Account for uncertainty
  - Used to evaluate expected values
- Example - Venture Capital Problem



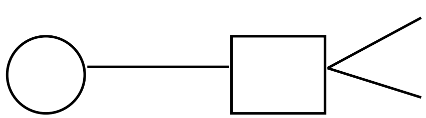
Decision Trees (continued)

- **Expected return on investment:**
  - If investment is made $E(I) = pR + (1 - p)L$
  - If investment not made $E(N) = r$
- **Decision:**
  - Invest if $pR + (1 - p)L > r$
  - Don't invest if $r > pR + (1 - p)L$

  What would you do if $r = pR + (1 - p)L$?
- **Decision Trees evaluated left to right**



decision must be made before uncertain event takes place



decision is conditional on the known outcome of the uncertain event

Example

Do I take my umbrella or not?

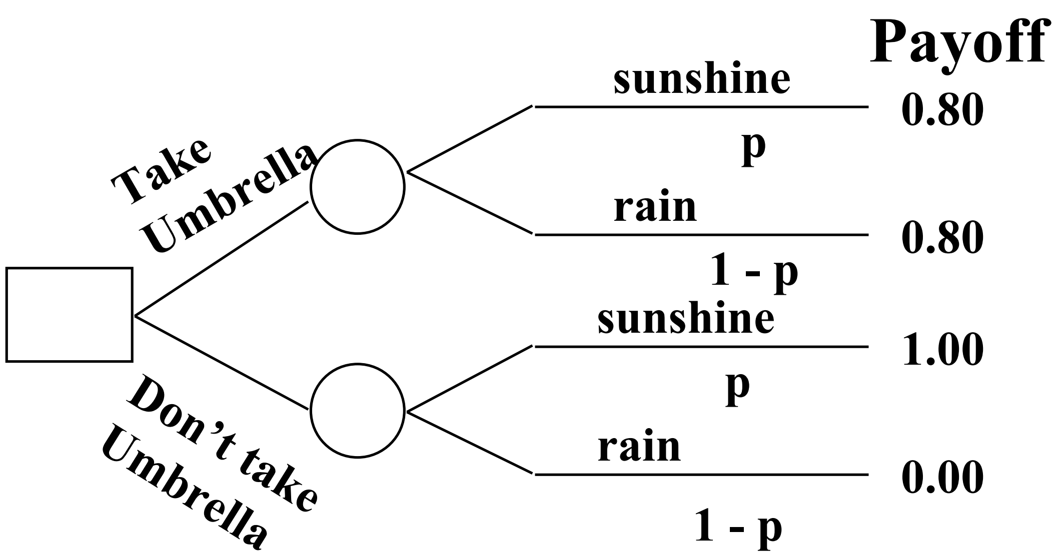
– If I don't and it is sunny, that is best - value 1.00

– If I do I won't get wet but it's inconvenient

  - value 0.80

– If I don't and it rains I ruin my suit - value 0.00

– Probability of sunshine is p



<u>Payoff Calculation</u>:

– Take umbrella

$0.8p + 0.8(1-p) = 0.8$

– Don't take umbrella

$1.0p + 0(1-p) = p$

Therefore:

Take umbrella if $p < 0.8$

Example 2

Texaco vs Pennzoil (1984)

- **Pennzoil and Getty agreed to merge**
- **Texaco made Getty a better offer - Getty reneges**
- **Pennzoil sues, wins case in 1985, get $11.1 Billion**
- **Texas appeals court reduces judgment by $2 Billion**
  - **With court costs and interest $10.3 Billion**
- **Texaco threatened to bankrupt and go to Supreme Court**
- **1987, before Pennzoil starts issuing liens Texaco offers to settle for $2 Billion**
- **Pennzoil thinks $3-5 Billion is a fair price**

What should Hugh Liedtke, CEO of Pennzoil, do?

Example 2 (continued)

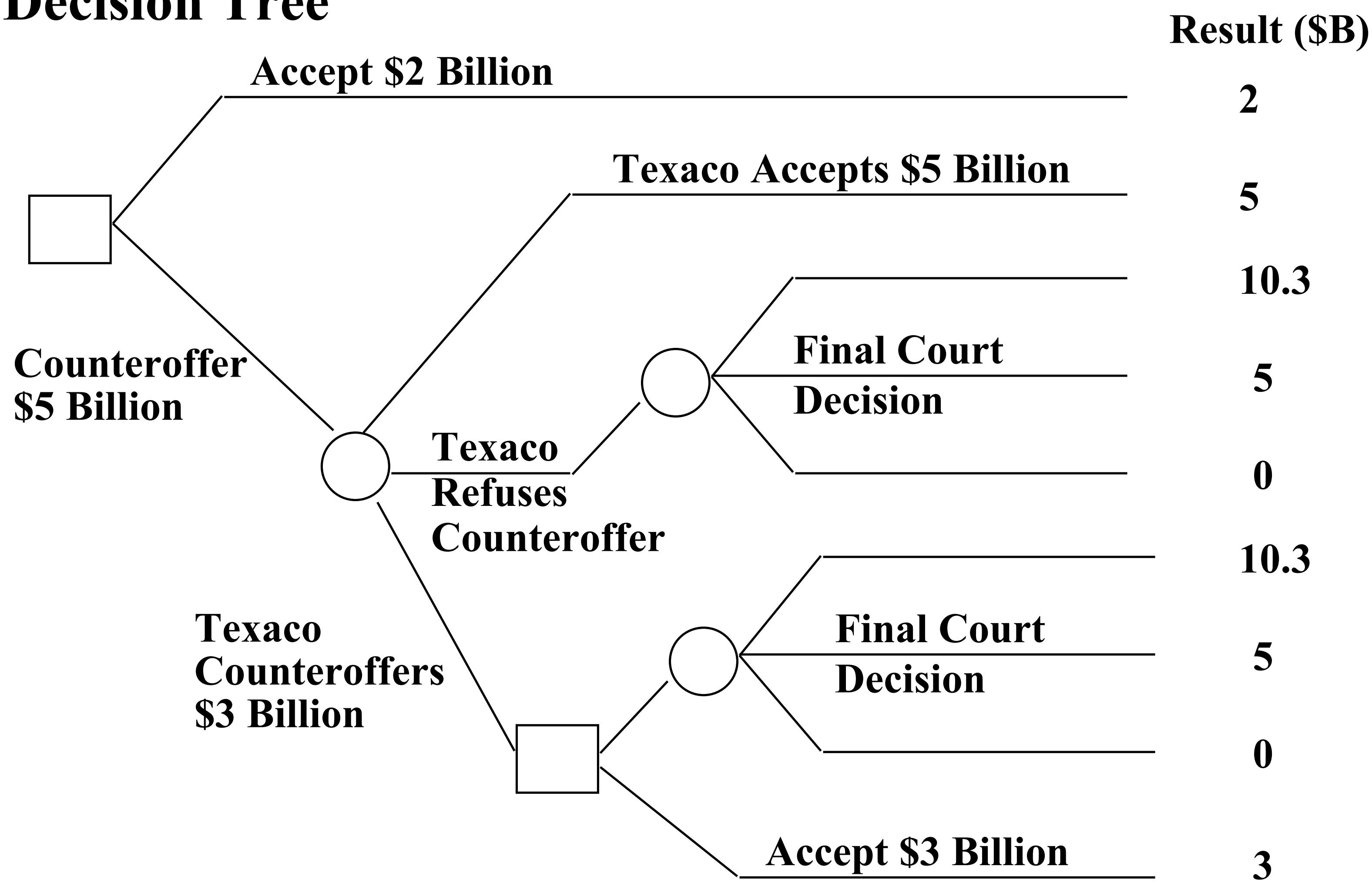
Options:

- **Accept $2 Billion**
- **Refuse and make counteroffer - risky**
  - **$5 Billion counteroffer**
    - **Texaco might accept**
    - **Texaco might counteroffer $3 Billion**
      - **Pennzoil might accept**
      - **Pennzoil might refuse (court)**
    - **Texaco might refuse counteroffer (court)**

Note: This is a simplified problem

Example 2 (continued)

Decision Tree



Example 2 (continued)

How could this tree be made more complex?

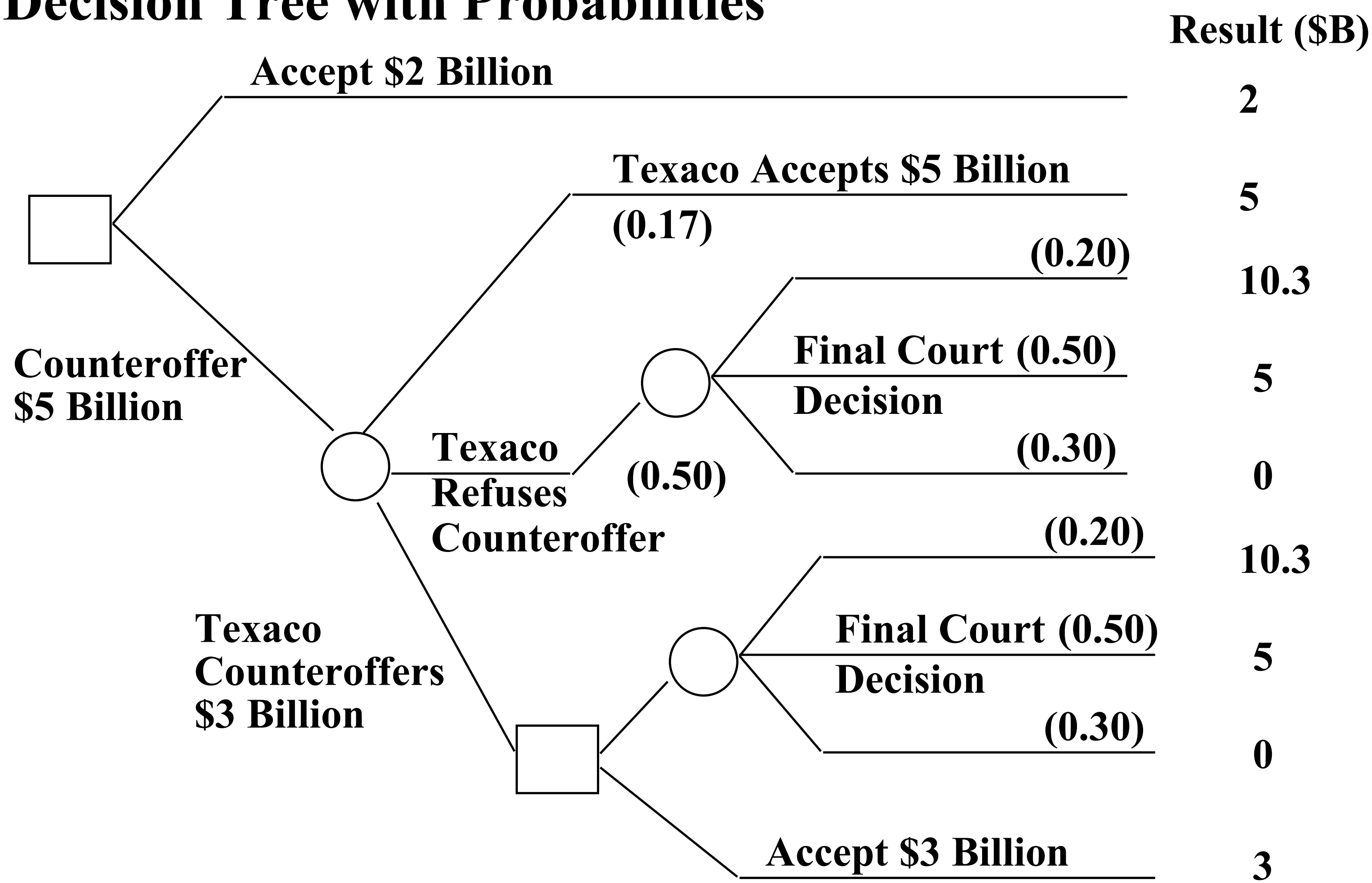
- **More options on size of initial counteroffer**
- **Other possible values of second counteroffer**
- **Other possible values of Texaco's counteroffer**
- **Other possible outcomes of court decision**
- **Include Texaco's bankruptcy option**

Probability Estimates

- **Even chance (50%) Texaco won't negotiate further**
- **\$5 B counteroffer twice as likely accepted as \$3 B (~33% for \$5 B, ~17% for \$3 B)**
- **Court Decision: 20% chance court will award \$10.3 B, 30% chance of no award → 50% for \$5 B**

Example 2 (continued)

Decision Tree with Probabilities

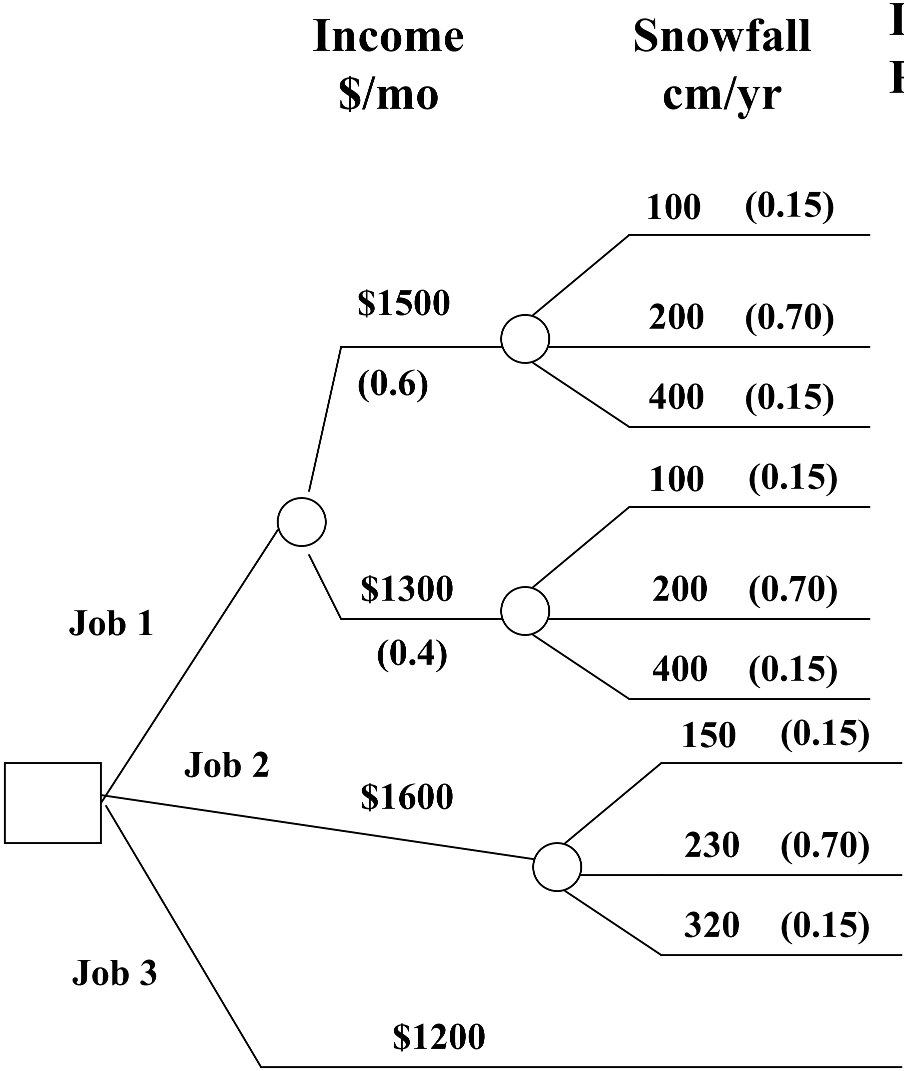


Multiple Objectives with Uncertainty

- **Single outcome (consequence) replaced by a vector of measures for all the objectives**
- **Use value function (or equivalent hierarchy decision tree) to reduce the measures to a single score**
- **Proceed as before**

Example: Robin gets 3 job offers in different places. 3 objectives are important: 1. Disposable income (amount left over after paying expenses); 2. Ability to participate in winter sports; 3. Quality of community. The attributes for the objectives are 1. Take home pay – rent; 2. Annual snowfall; 3. Results of magazine survey.

Decision Tree with Multiple Objectives (continued)



Income Rating (.50)	Snowfall Rating (.30)	Magazine Rating (.20)	Rating
75	25	56	56.2
75	50	56	63.7
75	100	56	78.7
25	25	56	31.2
25	50	56	38.7
25	100	56	53.7
100	37.5	0	61.25
100	57.5	0	67.25
100	80	0	74
0	0	100	20