

OR 542: Stochastic Models

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Purpose of the Course

- Provide a broad exposure to problems with important *Stochastic* components
- Introduce solution techniques for a variety of probabilistic applications
- Introduce computational aspects of stochastic modeling

Note: This course is broad but not deep. There is a complete course on each of the topics covered. This is just a start!

Definition of Operations Research

Operations Research (from Hillier and Lieberman)

- “ 1. The structuring of real life situation in a mathematical model, abstracting the essential elements so that a solution relevant to the decision maker can be sought
2. Exploring the structure of such solutions and developing systematic procedures for obtaining them
3. Developing a solution, including the mathematical theory, if necessary, that yield an optimal value of the system measure of desirability (or possibly comparing alternative courses of action by evaluating their measure of desirability). ”

Applying OR/MS

- Observation of system to be studied
- Problem definition
 - Identify objectives, goals, alternatives, measurement criteria
- Model construction
 - Identify relevant components and their interrelationships
- Problem solution
 - use various quantitative solution techniques
- Solution implementation
 - Feasibility, quality

Problem Identification

Fundamental Problem Types

- (1) Problems of *Association*
 - (2) Problems of *Construction*
 - (3) *Combinations* of the above two types
 - (4) Problems beyond our understanding
- } To be discussed later

- Tasks involving creativity
- Tasks needing leaps of faith - intuition
- Reasoning by (obscure) analogy
- Learning how to talk (language)

★ These problems are hard to define, characterize, model and solve

Problems of Association

- Association of attributes of given object(s) to specific outcome (class or response)
- These include:
 - *Prediction*
 - *Pattern classification or pattern recognition*
- Typically solved by Substitution:
 - Facts or data substituted into model
 - Outcome or outcomes are observed
- Examples
 - Curve fitting
 - Forecasting
 - Medical diagnosis
 - Trouble-shooting

Problems of Construction

- Involve need to create plan or configuration from among a set of alternatives
- These include:
 - Development of combinations
 - Blending problems
- Typically solved by Feedback and Comparison
 - Trial solution is found
 - Solution is compared to some criteria
 - Better solution is sought
- Examples
 - System configuration
 - Product mix
 - Facility location
 - Capital budgeting

Fundamental Operations

- Substitution and Inference
 - Insertion of information into model for evaluation
 - Interpret the output of the model
 - Feedback
 - Return of information regarding status of solution
 - Done periodically through solution process
 - Comparison
 - Comparing attributes of problem objects
 - Perhaps using the results to order the objects
- ★ Problems of Association need only *Substitution and Inference*
- ★ Problems of construction may need all

Message

- Know the enemy!
- Use the right model for the problem
 - Don't use what you have because you have it
 - Avoid hammer and nail syndrome
- Problem identification is the first step
- All problems can be categorized as association or construction or combination of the two
 - Break problem into its components
 - Categorize each piece
 - Right model for each

Modeling

Definition: *Model* - the representation of one system by another

- Can represent some theory or law
- Can account for empirical observations
- Combination of the two

Essence of MS - gain insight into real system by studying model

Model Building

- Involves abstraction and simplification
- Contains essential elements of the system
- Artistic

Properties of Models

- Organizes theoretical beliefs and empirical observations about the system
- Helps identify important aspects of system
- Allows testing of system
 - Response to different stimuli
 - Desirability of changes in the system
- Usually easier to manipulate than whole system
- Allows control over sources of variation
- Low cost (relatively)

Types of Models

- (1) Verbal Models - experts sit and discuss the system
 - Fast and low cost
 - Often ambiguous or biased - totally subjective
- (2) Physical Model or Iconic Model
 - Mock-up
 - Used frequently in engineering and physical sciences
- (3) Schematic or Analog Model
 - Property of real object is substituted by another object and used in similar manner
 - Map is an example

Types of Models

(4) Symbolic Model - usually mathematical

- Probability models
- Statistical models
- Optimization models
- Simulation models (can include aspects of the others)

Aspects to be Considered

(1) Families of Laws

- Indexed by parameters
- Not under human control
ex. Arrival rate of customers to a bank

(2) Families of Policies

- Indexed by factors
- Under human control
ex. Number of tellers in the bank

Classification of Models

- Prescriptive/Descriptive
 - Prescriptive used to formulate and optimize a system
 - Descriptive used to help understand the behavior of a system
- Discrete/Continuous
 - Continuous models have real valued variables
 - Discrete models don't
- Stochastic/Deterministic
 - Stochastic (probabilistic) includes random events
 - Deterministic models don't (usually expected value)

Classification of Models

(continued)

- **Static/Dynamic**
 - **Static models have variables that don't change over time (snapshot of the system in steady state)**
 - ex. **Evaluation of physical layout of a factory**
 - **Try several configurations**
 - **Evaluate performance of each**
 - **Dynamic include time dependent variables**
 - ex. **Queuing analysis of a bank during a whole day**
 - **Arrival rates change**
 - **Number of tellers changes**

Implementation

- **Optimal solutions may not be good management**
- **Solutions should be examined for reasonableness, intuition, etc.**
- **Results in more than a single value -- often a "policy" described in solution**
- **Communication of results is key**
 - **Charts and graphs make understanding easy and helpful**
- **Bottom line: Management science should support good decision making**
 - **Good management AND good mathematics**