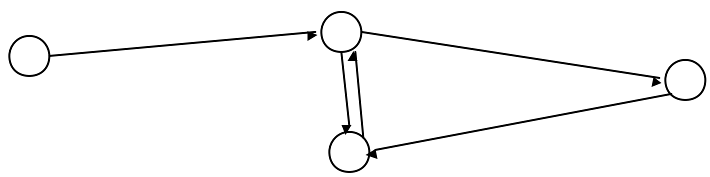
Networks

What is a "network"?

A set of nodes connected by a set of arcs (links)
Also called a "graph" in mathematics



Network Definitions

Directed Graph: Where each arc has a direction of travel associated with it. Essentially, a set of one-way streets.

Cyclic vs. Acyclic: A directed graph is cyclic if it contains any loops (i.e., can you get back to Start?)

Network Type Problems

Several linear/integer problems use network formulations

Trans-shipment problem

Shortest path or route

General network flow

Transportation/assignment problem

Network Components

- **Nodes**
  - **Represent locations, junction points, or states (conditions).**
  - **May be sources (generators) or sinks (destinations/consumers) of something.**
- **Arcs (edges)**
  - **Connect the nodes.**
  - **Usually *directed*.**
  - **A value is associated with the arc defining an arc-specific characteristic**

    (distance or travel time between nodes, cost of moving, etc.)

Network Component Details

Think of the network in action: "things" are generated at *supply nodes*, flow over the *arcs*, and are consumed at the *demand nodes*

Negative number = a supply at that node.

Positive number = a demand at that node.

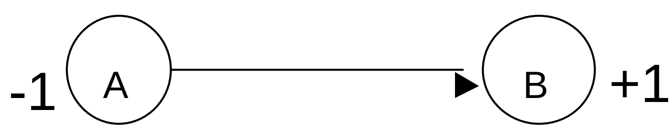
***Trans-shipment nodes*: can both send and receive.**

What kind of "things"?

Anything quantifiable.

The simplest network

One supply node, one arc, one demand node:



- One "thing" flows from A to B.

General Network Stuff

Things you probably realize:

- **Flows must balance through the network**
- **Arcs Û Decision Variables**
  - **The arc "cost" times how many things you decide to move along that arc.**
- **Nodes Û Constraints**
  - **How many things must be moved from or to specific nodes**
- **Generally, the objective function is to find the *minimum* cost solution (but depends on the aopplication)**
- **"Pure" network problems end up with integer solutions.**

  Why is this true?

New Spreadsheet functions

VLOOKUP(*value, range, column index, interpolate?*)

Search for a value in the leftmost column of a table, then return corresponding (same row) value from the specified column in the table

***value*: search in 1st column for entry closest to but not greater than. (identify the row)**

***range*: range for entire table**

***column index*: return value of cell in this column and identified row**

***interpolate*?: if missing or TRUE, use 1st column entry £ lookup value. If FALSE, must find exact match or else will return "#N/A"**

VLOOKUP notes

1st arguments that are too small will result in #N/A being returned

HLOOKUP provides a similar function, going horizontal instead of vertical

Column index (3rd arg.) must be within RANGE

LOOKUP & INDEX also may be useful from time to time

Example 1

density	viscosity	temperatur
0.457	3.55	500
0.525	3.25	400
0.616	2.93	300
0.675	2.75	250
0.746	2.57	200
0.835	2.38	150
0.946	2.17	100
1.09	1.95	50
1.29	1.71	0

VLOOKUP()

assume "R" = table coords.

(1, R, 1, TRUE) = 0.946

(1, R, 2) = 2.17

(1, R, 3, T) = ?

(0.746, R, 3, F) = ?

(0.1, R, 2, T) = ?

Second new function

SUMIF(*range, criteria, range to sum*)

for elements in *range* that meet *criteria,* add corresponding elements in *range to sum* and return

	A	B
1	100	7
2	200	14
3	300	21
4	400	28

SUMIF(A1:A4,">160",B1:B4)= ?

SUMIF(A1:A4,"<200",B1:B4)= ?

The Trans-shipment Problem

Objective: Determine how goods should flow through the network to meet stated demands with available supply at minimum cost

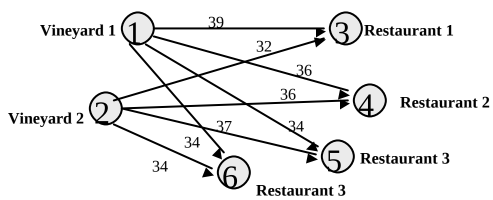
Each arc has a cost, each node has a supply or demand

If no supply or demand is specified for a node, then "flow in" = "flow out" (conservation of mass)

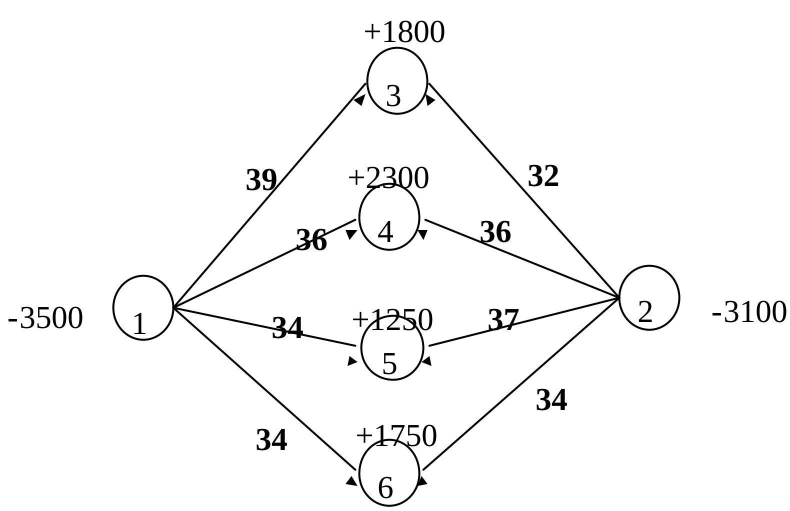
The Wine Problem

- Two source nodes (the vineyards)
- Four demand nodes (the restaurants)
- Very limited set of connecting arcs
- Each arc has an associated shipping cost per case of wine

- We want to find the least cost method of meeting the demand for wine at all the restaurants



The Wine Problem as Trans-shipment



Solving in Excel as LP

List each from-to pair (arc) with distance/cost

VLOOKUP function labels ends of each arc

Arc variables indicate which arcs are chosen

Remember: don't need to specify integer or binary variables---supply/demand of -1/+1 will do

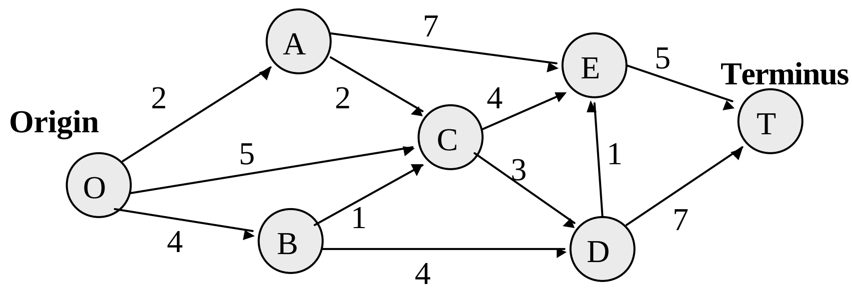
Net flow calculation keeps track of routing

Edit supply (-1) & demand (+1) settings to solve for different pairs in network

Shortest Route Problem

- Objective: Determine the shortest distance between an originating point and a destination point through a network
- Like transshipment problem with supply/ demand of 1 at origin/destination
  - Remaining nodes are transshipment nodes, and flow in must equal flow out
- Can be solved either as a linear programming problem or by special algorithm

Shortest Path Example



Labeling Algorithm for Shortest Path

Dikstra's Algorithm - finds shortest path between one node and all the others that have a path from the starting node

Algorithm:

Step 0: Label the starting node (the origin) as 0*

(note that the * means that the label is permanent)

Label all the other nodes, i, with temporary labels, $l_i = d_{0i}$ for all i = 1, …, n. (If there is no direct arc between 0 and i then $d_{01} = \infty$)

Step 1: Choose smallest of temp labels, $l_k = \min_{i} [l_i]$

and make it permanent (l_k*)

Step 2: For node V_k (just labeled), replace all labels for neighbors of V_k by the following rule $l_i = \min[l_i, l_k^* + d_{ki}]$

Return to Step 1 or stop if all nodes are permanently labeled

Network Flow Problem

Objective: Maximize the total amount of flow from an origin to a destination.

Each arc now has a limited flow capacity (upper bound)

Like transshipment problem if return arc added from destination to origin

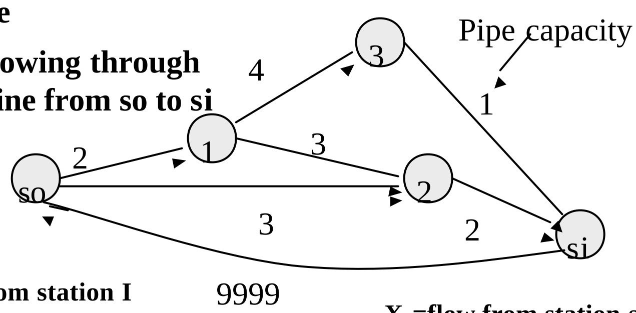
Maximize flow across return arc

Supply/demand at *each node* of 0 (balanced)

Solving in Excel

- List each from/to pair with arc capacity
- Add (fake) return arc with unlimited (e.g., 9999) capacity
- Net flow must be zero across all nodes
- Maximize cell set equal to flow across return arc
- Example

Oil flowing through pipeline from so to si



X_{ij}=flow from station I to station j

X_0=flow from station si to station so

Transportation/Assignment Problem

Objective: Same as in transshipment problem (minimize cost of satisfying demand from various sources) but with no transshipment nodes

- Simpler problem structure allows simpler solution procedure

Assignment problem has demands and supplies of 1

- Assigning jobs to machines, people to offices, etc.

Solving in Excel

Variables set up in a matrix

- Demand nodes in columns
- Supply nodes in rows

Second matrix reflects costs of each route

- Objective function uses sumproduct function

Rows (supply) have upper limits

- $\leq$ constraints

Columns (demand) have minimum requirements

- $\geq$ constraints

Minimal Spanning Tree Problem

Objective: Connect all the nodes in the network so that the total arc lengths are minimized

Iterative (constructive) algorithm provides optimal solution

Not solvable as LP---easy manual algorithm

Minimal Spanning Tree Solution Method

Select any starting node

Select the node closest to the starting node (in arc length/cost)

Select the closest node not presently in the spanning tree

Repeat last step until all nodes are in the spanning tree

Minimal Spanning Tree

Not solvable using Excel and our current software

Manual algorithms presented in class

- **Similar to a “greedy” algorithm**

The Traveling Salesman Problem

Given a set of cities and travel costs between them. One city is designated as Start/Finish. Travel costs are symmetric (i.e., cost(AB) = cost (BA).

Salesman Bill must visit all the cities and return home. The order is up to Bill.

Minimize his total travel cost.

http://www.keck.caam.rice.edu/tsp/