**OR 542 Stochastic Models**

**Problem Set #1 - Probability Review**

1. If we draw one card from a 52 card deck, what is the probability that it is red? That it is a diamond? That it is an ace? That it is the ace of diamonds?

2. A pair of dice is rolled once. Compute the probability that the sum is equal to each of the integers 2 through 12.

3. A fair coin is flipped 4 times. What is the probability that the fourth flip is heads given that each of the first 3 flips resulted in heads?

4. For a particular gambling game in which you bet $1, the following are the payoffs you could get, together with their associated probabilities:

| Payoff | Probabilities |
|---|---|
| - 1 | 125/216, |
| 1 | 75/216, |
| 2 | 15/216, |
| 3 | 1/216. |

Compute the expected payoff, and the variance of the payoff.

5. Using the results of problem 2, compute the expected sum of a single roll of 2 fair dice.

6. Box 1 contains 4 defective and 16 nondefective light bulbs. Box 2 contains 1 defective and 1 nondefective light bulb. We roll a fair die 1 time. If we get a 1 or a 2, then we select a bulb at random from box 1. Otherwise we select a bulb from box 2. What is the probability that the selected bulb will be defective?

7. Suppose there is a medical test that diagnoses a particular disease with 95% accuracy on both those that do have the disease and on those that do not have it. If 0.5% of the population actually has the disease, compute the probability that a particular individual has the disease, given that the test says he does.

8. Compute the probability that in a room containing n people, that at least 2 will have the same birthday. Do this for n = 10, 20, 30, 50. (This will be covered in class).

9. Find the CDF of the exponential distribution given the density function is $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$.

10. Find the probability of winning the lottery given that you must pick 6 numbers from 1- 48. Assume that there is only one winner.

11. Consider the following probability density function: $f_X(t) = 1/5$, $5<t<10$ and 0 otherwise.

Compute the variance of this distribution in two ways:

1. Using $Var[X] = E[(X-\mu_x)^2]$ directly

2. Using the computational formula $Var[X] = E[X^2] - \mu_x^2$