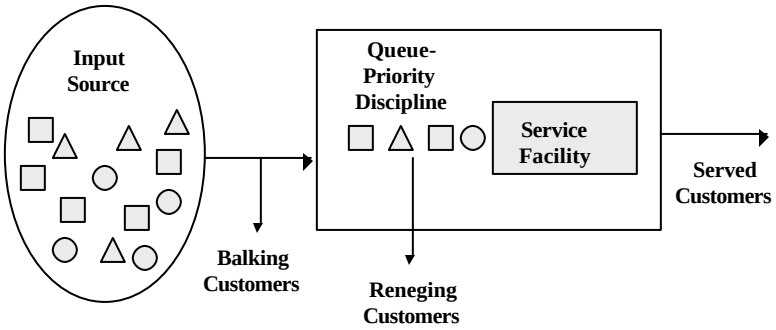
Queueing Theory

- **Definition: The mathematical study of waiting lines**
- **Structure of Queueing Systems**



Queueing Theory

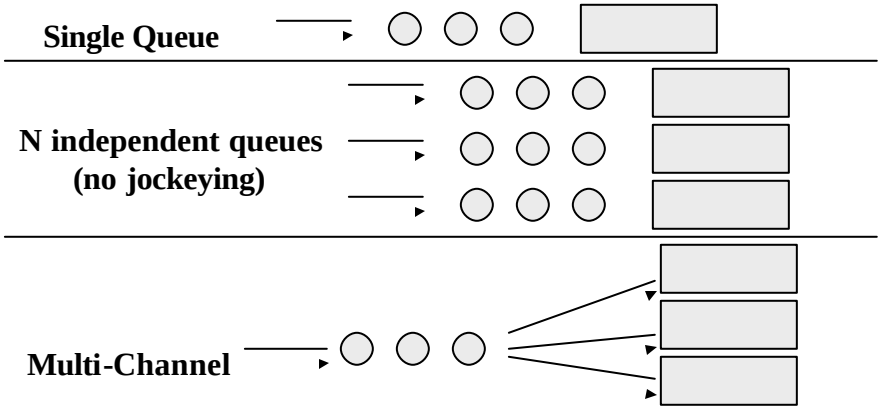
- **Queueing Models**
  - **Different ones for different situations**
  - **Limited by very stringent assumptions**
  - **Cannot handle complex problems**
    - **Approximations**
    - **Simulation**
- **Queueing System Characteristics**
  - **Arrival Process**
  - **Service Process**
  - **Number of Channels**
  - **Number of Phases**
  - **Queue Discipline - Selection for Service**

Arrival Process

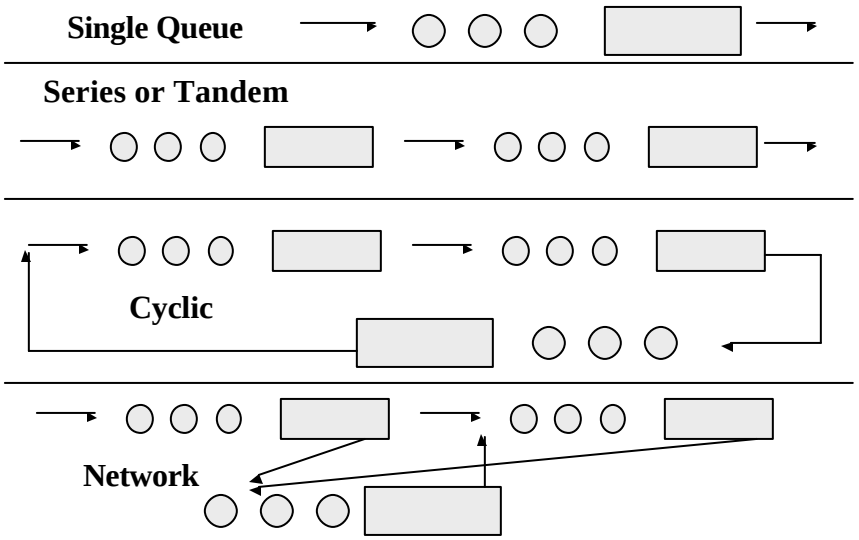
- **Probability Distribution of Arrivals over time**
  - **Usually Poisson Process**
    - **Number of Arrivals in time interval ~ Poisson**
    - **Time between arrivals ~ Exponential**
    - **Memoryless Property**
  - **Deterministic**
  - **General Distribution**
- **Single or simultaneous arrivals (batch or bulk)**
- **Behavior of arrivals**
  - **Impatience (Balking or Reneging)**
  - **Jockeying**
- **Time behavior of system (Stationary or not)**
- **Capacity limits**

Service Process

- **Probability Distribution of Service time**
  - **Usually modeled as Exponential**
  - **General**
- **Single or simultaneous service (bulk or individual)**
- **Number of Parallel service channels**



Number of Stages or Phases



Selection for Service

- **First com first served (FCFS or FIFO)**
- **Last in First out (LIFO)**
- **Random**
- **Priority**
  - **Preemptive**
    - **Resume**
    - **Restart**
  - **Non-Preemptive**

Notation

A/B/x/y/z

- **A = letter for arrival distribution (e.g., m, e, g, d, …)**
- **B = letter for service distribution**
- **x = number of service channels**
- **y = number allowed in queue (truncation)**
- **z = queue discipline (e.g., reneging)**

Examples of Queueing Systems

- **Commercial Service Systems (outside customers)**
  - **Barber shop**
  - **Bank teller service**
  - **Cafeteria Line**
- **Transportation Systems**
  - **Traffic light**
  - **Airplanes waiting to land or take off**
  - **Taxi cabs, Fire trucks, etc.**
- **Business or Industrial Systems**
  - **Material Handling Systems**
  - **Machine repair**
  - **Production lines**
- **Social service Systems**
  - **Judicial System**
  - **Health Care System**

Measurement of System Performance

- **Descriptive not Prescriptive**
  - **Don't find optimal design**
  - **Describe and the analyze the system**
  - **Compare alternatives**
- **Measures used:**
  - **System size**
    - **Number in queue**
    - **Number in system**
  - **Customer waiting times**
    - **Time in queue**
    - **Time in system**
  - **Server idleness**

Single Server System (m/m/1) Assumptions

- **Input Source: Infinite with no balking or reneging**
- **Arrival Distribution: Poisson with rate** λ
- **Service Distribution: Exponential with rate** μ
- **Queue: Single line, unlimited length**
- **Number of servers: 1**
- **Number of Phases: 1**
- **Selection for Service: FIFO**