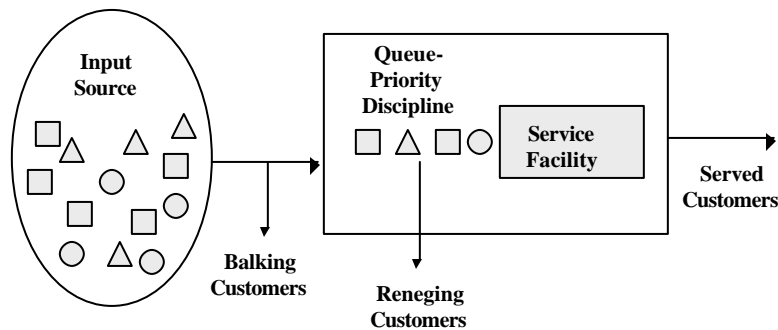


Queueing Theory

- **Definition:** The mathematical study of waiting lines
- **Structure of Queueing Systems**



Queueing Theory

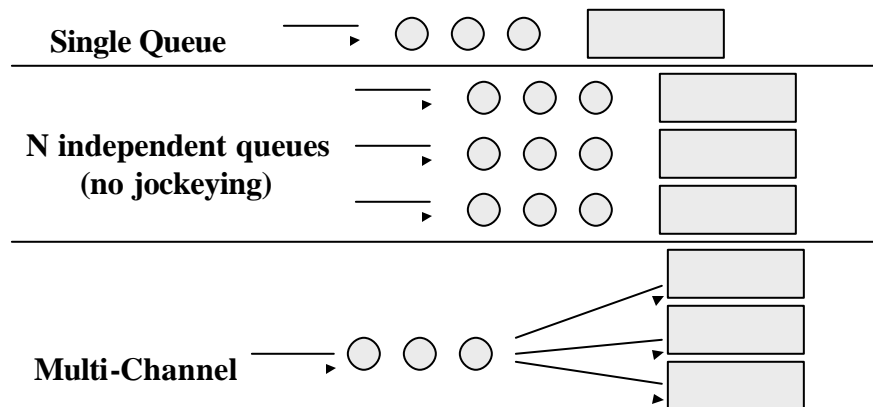
- **Queueing Models**
 - Different ones for different situations
 - Limited by very stringent assumptions
 - Cannot handle complex problems
 - Approximations
 - Simulation
- **Queueing System Characteristics**
 - Arrival Process
 - Service Process
 - Number of Channels
 - Number of Phases
 - Queue Discipline - Selection for Service

Arrival Process

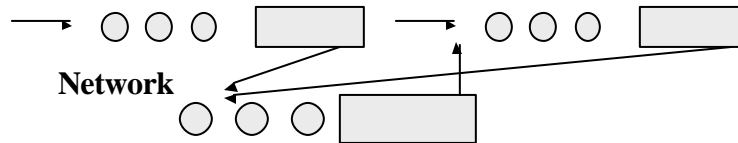
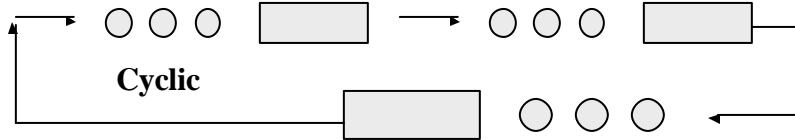
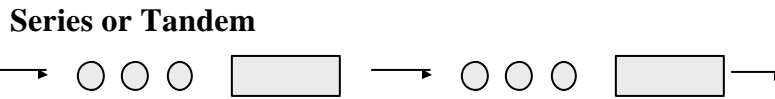
- **Probability Distribution of Arrivals over time**
 - Usually Poisson Process
 - Number of Arrivals in time interval ~ Poisson
 - Time between arrivals ~ Exponential
 - Memoryless Property
 - Deterministic
 - General Distribution
- **Single or simultaneous arrivals (batch or bulk)**
- **Behaviors of arrivals**
 - Impatience (Balking or Reneging)
 - Jockeying
- **Time behavior of system (Stationary or not)**
- **Capacity limits**

Service Process

- **Probability Distribution of Service time**
 - Usually modeled as Exponential
 - General
- **Single or simultaneous service (bulk or individual)**
- **Number of Parallel service channels**



Number of Stages or Phases



Selection for Service

- **First com first served (FCFS or FIFO)**
- **Last in First out (LIFO)**
- **Random**
- **Priority**
 - **Preemptive**
 - **Resume**
 - **Restart**
 - **Non-Preemptive**

Notation

$A/B/x/y/z$

- **A** = letter for arrival distribution (e.g., m, e, g, d, ...)
- **B** = letter for service distribution
- **x** = number of service channels
- **y** = number allowed in queue (truncation)
- **z** = queue discipline (e.g., reneging)

Examples of Queueing Systems

- **Commercial Service Systems (outside customers)**
 - Barber shop
 - Bank teller service
 - Cafeteria Line
- **Transportation Systems**
 - Traffic light
 - Airplanes waiting to land or take off
 - Taxi cabs, Fire trucks, etc.
- **Business or Industrial Systems**
 - Material Handling Systems
 - Machine repair
 - Production lines
- **Social service Systems**
 - Judicial System
 - Health Care System

Measurement of System Performance

- **Descriptive not Prescriptive**
 - Don't find optimal design
 - Describe and the analyze the system
 - Compare alternatives
- **Measures used:**
 - **System size**
 - Number in queue
 - Number in system
 - **Customer waiting times**
 - Time in queue
 - Time in system
 - **Server idleness**

Single Server System (m/m/1) Assumptions

- **Input Source: Infinite with no balking or renegeing**
- **Arrival Distribution: Poisson with rate λ**
- **Service Distribution: Exponential with rate μ**
- **Queue: Single line, unlimited length**
- **Number of servers: 1**
- **Number of Phases: 1**
- **Selection for Service: FIFO**