ARE 155, Week 1
A Simple Notation Example

Here is a fairly informal example to help you understand the notation of the basic model. Rather than writing down the model and then interpreting the notation in terms of an example, we'll start with an example, then translate it into the linear programming notation.

The problem we face is to make as much money as we can selling cookies, using only the ingredients we have on hand. We can make either Toll House (chocolate chip) cookies or oatmeal cookies, and we need to decide how many batches of each to make. We get \$ 20 a batch for chocolate chip cookies, and \$ 10 a batch for oatmeal cookies. The recipes for each follow:

Toll House Cookies

1 C brown sugar	*1/2 C white sugar*
1 C butter	*2 eggs*
1 t vanilla	*1 t soda*
1 t salt	*2 1/4 C flour*
1 bag chocolate chips	

Oatmeal Cookies

1 C brown sugar	*1/2 C white sugar*
1 C butter	*2 eggs*
1 t vanilla	*1 t soda*
1/2 t salt	*1 1/2 C flour*
1 C raisins	*3 C oatmeal*
1 t cinnamon	

We have the following ingredients on hand:

5 C brown sugar
5 C white sugar
1 4 oz. bottle vanilla (24 t)
1 dozen eggs
1 box soda (64 t)
1 box salt (64 t)
1 bag flour (18 1/4 C)
1 box oatmeal (15 C)
2 bags chocolate chips
5 C raisins
1 bottle cinnamon (32 t)

The challenge is to set this up as a linear programming problem. (We won't solve it today!) First, let's set up the objective function Z. Our problem is to maximize revenue from selling batches of toll house (TH) and batches of oatmeal (OM) cookies:

$$\max_{\text{TH, OM}} \quad Z = \text{ rev. from TH} * \text{batches of TH} + \text{rev. from OM} * \text{batches of OM}$$

Before we move on, let's identify the *parameters* ("uncontrollable inputs") and *decision variables* ("controllable inputs") in our objective function. The *parameters* are the information that we already know, and the *decision variables* are the information that we want to figure out. So, in this problem we know the revenues we get from selling each kind of cookie, and we want to figure out how many batches of each to produce.

We use c_j to represent cost, revenue, or profit parameters. In this problem we have two:

$$c_1 = \$20 \; \frac{\text{dollars}}{\text{batch of TH}} \quad c_2 = \$10 \; \frac{\text{dollars}}{\text{batch of OM}}$$

We also have two decision variables:

$$x_1 = \text{batches of TH} \quad x_2 = \text{batches of OM}$$

So, our objective function can be written:

$$\max_{\underbrace{TH}_{x_1},\underbrace{OM}_{x_2}} \quad Z = \underbrace{20 \quad \frac{\text{dollars}}{\text{batch of TH}}}_{c_1} * \underbrace{\text{batches of TH}}_{x_1} + \underbrace{10 \quad \frac{\text{dollars}}{\text{batch of OM}}}_{c_2} * \underbrace{\text{batches of OM}}_{x_2}$$

Notice that the units cancel – what then, are the units of our objective function Z?

Moving along, we need to deal with the fact that we are limited by the ingredients we have on hand. These are our *constraints*. The constraints will also contain both parameters and decision variables. The first set of parameters in the constraint set define our input requirements for each batch of cookies. We represent these by $a_{i,j}$, where i refers to the number of the constraint, and j refers to the number of the decision variable. Our first constraint, b_1, is brown sugar. There is a coefficient for each type of cookie (they happen to be the same!):

$$a_{1,1} = 1 \; \frac{\text{cup}}{\text{batch}} \quad a_{1,2} = 1 \; \frac{\text{cup}}{\text{batch}}$$

Let's take an example where they are not the same, cinnamon. This is our 11th constraint. What do you think the value of $a_{11,1}$ will be?

$$a_{11,1} = 0 \; \frac{\text{t}}{\text{batch}} \quad a_{11,2} = 1 \; \frac{\text{t}}{\text{batch}}$$

Notice in both of these cases that the units of the $a_{i,j}$ coefficients are the same in each constraint, but different for different constraints. Finally, our last set of parameters reflect the resource limits for our problem. These are the b_i, where i refers to the constraint number. Here, we are limited by the ingredients on hand.

$$b_1 = 5C \text{ brown sugar}$$
$$\vdots$$
$$b_{11} = 32t \text{ cinnamon}$$

Now, let's put these coefficients together with our decision variables to form the constraints:

$$\underbrace{1\,\frac{\text{cup}}{\text{batch}}}_{a_{1,1}} * \underbrace{TH}_{x_1} + \underbrace{1\,\frac{\text{cup}}{\text{batch}}}_{a_{1,2}} * \underbrace{OM}_{x_2} \leq \underbrace{5C \text{ brown sugar}}_{b_1} \qquad \text{Constraint 1}$$

$$\vdots \qquad\qquad \text{you fill these in}$$

$$\underbrace{0\,\frac{\text{t}}{\text{batch}}}_{a_{11,1}} * \underbrace{TH}_{x_1} + \underbrace{1\,\frac{\text{t}}{\text{batch}}}_{a_{11,2}} * \underbrace{OM}_{x_2} \leq \underbrace{32t \text{ cinnamon}}_{b_{11}} \qquad \text{Constraint 11}$$

Again, notice how the units cancel. Filling in the units is a good way to make sure you are setting up the problem correctly – the units on both sides of the equation must match. A couple points for you to think about for this problem:

- In 1A or 100A, what did we call the type of production technology that we see here?
- Does this particular problem meet all of the 7 assumptions of the general linear model?
- Do you think you will choose to make nothing but chocolate chip cookies? Why or why not?